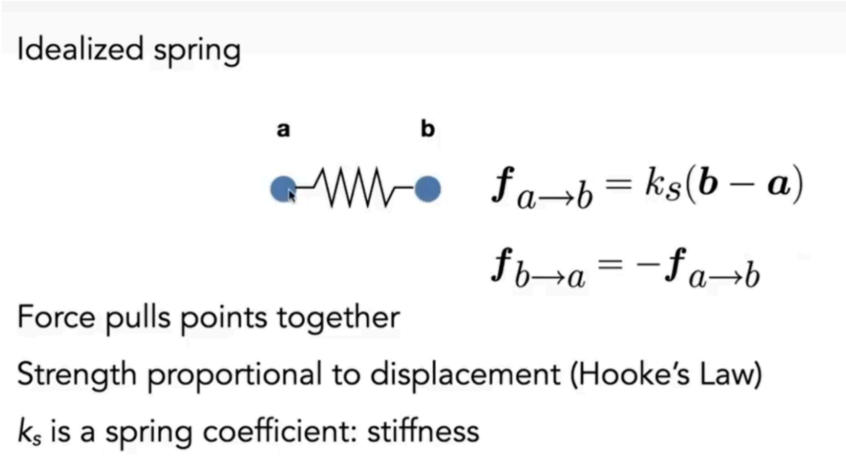
这节随便听听，再做一下作业 8

Final Project 开个开源库。

嗯哼，动画。

Mass Spring System 质点弹簧系统



改变 k_s，就是对不同材质的模拟，一种材质内部可能有不同的 k_s

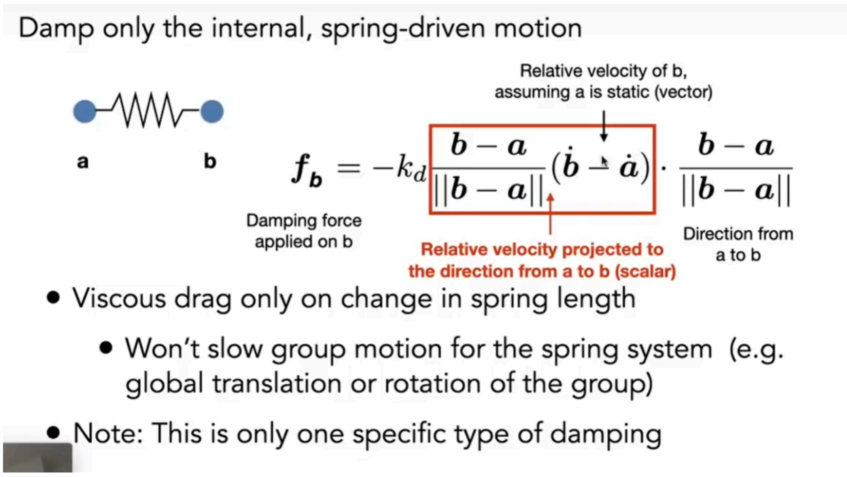


图 1　还要考虑摩擦力，否则系统不会停止运动

Particle System 粒子系统

说了挺多哈哈，懒得记了。

逆运动学

难题

1. 难解

2. 解不唯一

前向欧拉格式

Euler's Method

Euler's Method (a.k.a. Forward Euler, Explicit Euler)

- Simple iterative method
- Commonly used
- Very inaccurate
- Most often goes unstable

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \Delta t\, \dot{\boldsymbol{x}}^t$$
$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^t + \Delta t\, \ddot{\boldsymbol{x}}^t$$

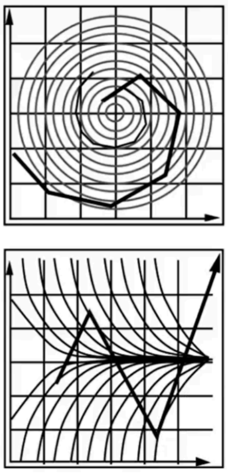
Instability of the Euler Method

The Euler method (explicit / forward)

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \Delta t\, \boldsymbol{v}(\boldsymbol{x}, t)$$

Two key problems:

- Inaccuracies increase as time step Δt increases
- Instability is a common, serious problem that can cause simulation to diverge



Witkin and Baraff

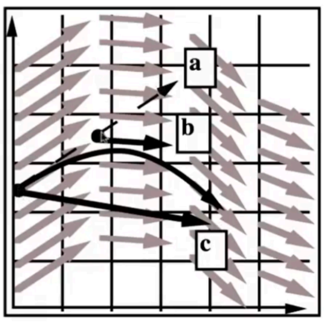
问题：除了采样率之外，误差会被无限放大才是更致命的。

Midpoint Method

Midpoint method

- Compute Euler step (a)
- Compute derivative at midpoint of Euler step (b)
- Update position using midpoint derivative (c)

$$x_{\text{mid}} = x(t) + \Delta t/2 \cdot v(x(t), t)$$
$$x(t + \Delta t) = x(t) + \Delta t \cdot v(x_{\text{mid}}, t)$$



Witkin and Baraff

Modified Euler

Modified Euler

- Average velocity at start and end of step
- Better results

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \frac{\Delta t}{2}\left(\dot{\boldsymbol{x}}^t + \dot{\boldsymbol{x}}^{t+\Delta t}\right)$$
$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^t + \Delta t\, \ddot{\boldsymbol{x}}^t$$

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \Delta t\, \dot{\boldsymbol{x}}^t + \frac{(\Delta t)^2}{2}\, \ddot{\boldsymbol{x}}^t$$

Adaptive Step Size

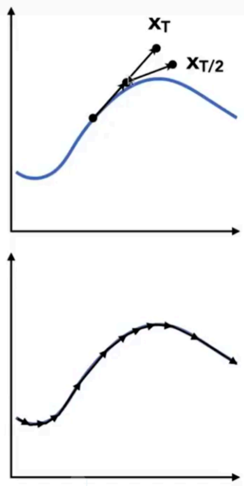
既然反正都要算这个……

Adaptive step size

- Technique for choosing step size based on error estimate
- Very practical technique
- But may need very small steps!

Repeat until error is below threshold:

- Compute x_T an Euler step, size T
- Compute $x_{T/2}$ two Euler steps, size T/2
- Compute error $\| x_T - x_{T/2} \|$
- If (error > threshold) reduce step size and try again

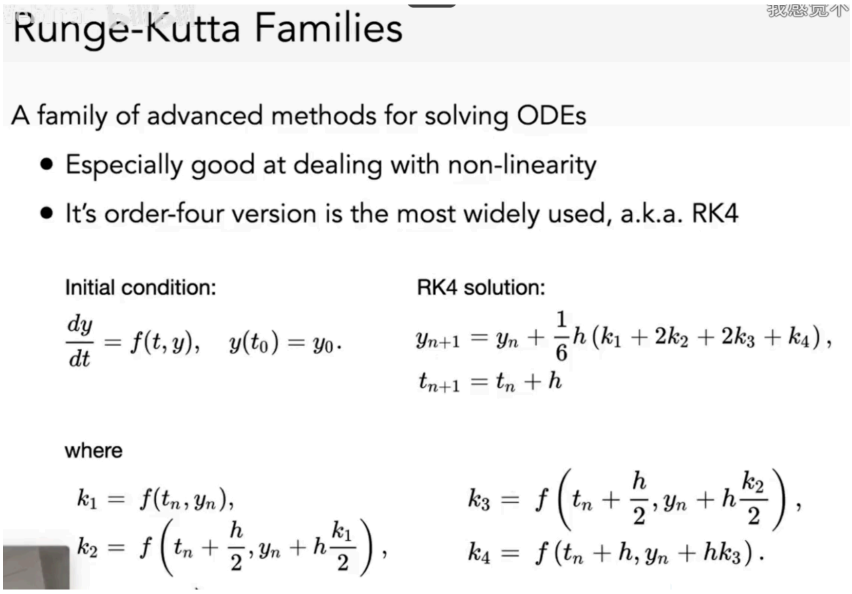


How to determine / quantize "stability"?

- We use the local truncation error (every step) / total accumulated error (overall)
- Absolute values do not matter, but the orders w.r.t. step
- Implicit Euler has order 1, which means that
  - Local truncation error: $O(h^2)$ and
  - Global truncation error: $O(h)$ (h is the step, i.e. Δt)

图 2 梦回渐进理论

Runge-Kutta



其实就是高级一点的中点法

完结撒花。

做一下作业 8

嗯，环境有问题，那就不做了。

后续的 final project，也是我带专生涯的毕业设计，会继续更新，放在 https://gitee.com/isirin1131_admin/specialized-graduation-program

告一段落喽。

有什么感言呢？我只是感觉有些无聊了，完成我的生成函数手稿后，就会开始 games001 的学习。

之后就是 games 200 系列了？或许会开始搞 AI 了，但那太远了，还是关注在短期目标上吧。